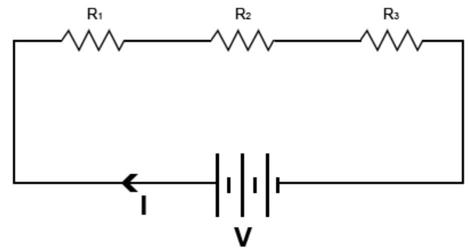
1317

ELECTRICAL CIRCUIT THEORY

NOTE: Each Question carries 10 marks.

1(a). Derive the equivalent resistance when three resistors are connected (i) in series (ii) in parallel. **[Each - 5 marks]**

RESISTANCE IN SERIES:



3 Resistors in series

- Resistances R_1, R_2, and R_3 are connected in series.
- Let V - be the applied voltage to the circuit
  I - be the total circuit current
- In a series circuit, the current through all resistors are same.
- Applied voltage is the sum of the individual voltage drops across each resistor.
- The total voltage across all three resistors is the sum of the voltage drops:

  $V = V_1 + V_2 + V_3$ --------------- (1)

- Using Ohm's Law (V= IR) for each resistor:
  - $V_1 = IR_1$ $V_2 = IR_2$ $V_3 = IR_3$

- Substituting in equation 1,

  $V = IR_1 + IR_2 + IR_3$

  $V = I (R_1 + R_2 + R_3)$

  $V / I = R_1 + R_2 + R_3$

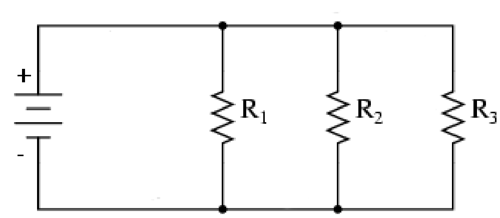
  $\mathbf{R = R_1 + R_2 + R_3 \ \Omega}$

  Where, R = V / I = Equivalent resistance

Thus, the Equivalent resistance for three resistors in series is the sum of their individual resistances.

RESISTANCE IN PARALLEL:

- Resistances R_1, R_2, and R_3 are connected in parallel.
- Let, V - be the applied voltage to the circuit
  I - be the total circuit current



3 Resistors in parallel

- I_1, I_2 and I_3 are the current through R_1, R_2 and R_3 respectively.
- **In parallel connection, the p.d. across all the resistors is the same but the current in each is different.**
- The total current in the circuit is the sum of currents flowing through each resistance.
  i.e., $I = I_1 + I_2 + I_3$ ----- (1)

- By Ohms law,
  $I_1 = V / R_1$, $I_2 = V / R_2$ and $I_3 = V / R_3$

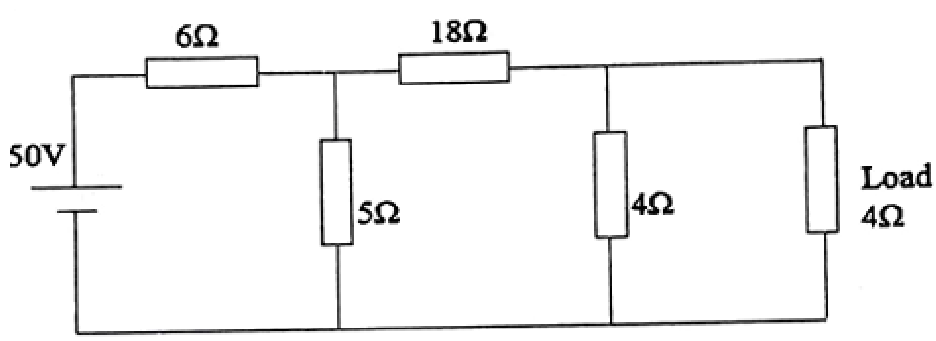
- Substituting in equation 1,
  $I = (V / R_1) + (V / R_2) + (V / R_3)$
  $I = V [(1/ R_1) + (1/ R_2) + (1/ R_3)]$
  $I / V = 1/R1 + 1/R2 + 1/R3$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eq} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Where, **R_{eq} is the equivalent resistance of parallel combination**.

1(b). Find the current in the 4 Ω load resistor in the circuit shown below by mesh analysis. [10 marks]

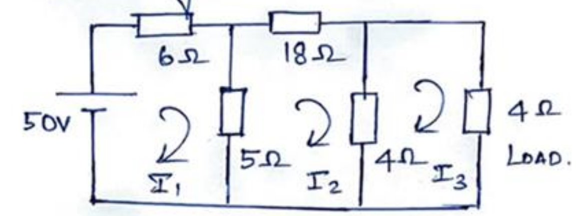


SOLUTION:

Let:

- I_1: Current in Mesh 1.
- I_2: Current in Mesh 2.
- I_3: Current in Mesh 3.

Assuming Mesh currents,



writing By Matrix form,

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} (6+5) & -5 & 0 \\ -5 & (5+18+4) & -4 \\ 0 & -4 & (4+4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 11 & -5 & 0 \\ -5 & 27 & -4 \\ 0 & -4 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

$$[R] \times [I] = [V].$$

$$\Delta = \begin{vmatrix} 11 & -5 & 0 \\ -5 & 27 & -4 \\ 0 & -4 & 8 \end{vmatrix} = 11\left[(27\times 8)-(16)\right] - (-5)\left[-40-0\right] + 0\left[\ \ \right]$$

$$= 11\,[216-16] + 5\,[-40] = 11 \times 200 - 200 = 10 \times 200$$

$$\boxed{\Delta = 2000}$$

$$\Delta_3 = \begin{vmatrix} +11 & -5 & 50 \\ -5 & 27 & 0 \\ 0 & -4 & 0 \end{vmatrix} = +50\,[20] = 1000 \Rightarrow \boxed{\Delta_3 = 1000}$$

Now,

$\Delta = 2000$

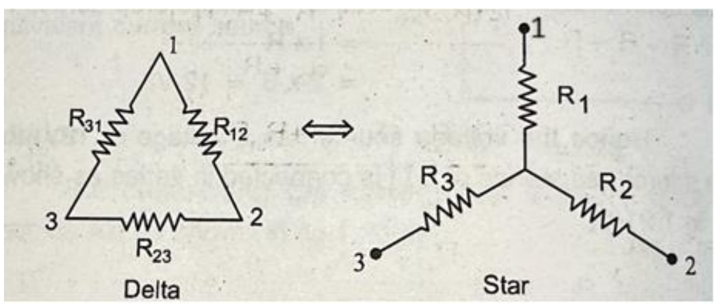
$\Delta_3 = 1000$

Current $I_3 = \frac{\Delta_3}{\Delta} = \frac{1000}{2000} = 0.5\ A.$

So, current through $4\,\Omega$ Load Resistor, $\boxed{I_3 = 0.5\ A}$.

1(c). Derive an expression for delta to star transformation. **[10 marks]**

SOLUTION:



As shown in the figure, if the two systems are equivalent then the resistance between points 1 and 2 in delta and Resistance between points 1 and 2 in star are equal.

Resistance between terminals 1 and 2:

- In the Δ network:

$$R_{12(\Delta)} = R_{12} + \frac{R_{31} \cdot R_{23}}{R_{31} + R_{23}}$$

- In the Y network:

$$R_{12(Y)} = R_1 + R_2$$

Equating the two:

$$R_1 + R_2 = R_{12} + \frac{R_{31} \cdot R_{23}}{R_{31} + R_{23}} \quad (1)$$

Resistance between terminals 2 and 3:

- In the Δ network:

$$R_{23(\Delta)} = R_{23} + \frac{R_{12} \cdot R_{31}}{R_{12} + R_{31}}$$

- In the Y network:

$$R_{23(Y)} = R_2 + R_3$$

Equating the two:

$$R_2 + R_3 = R_{23} + \frac{R_{12} \cdot R_{31}}{R_{12} + R_{31}} \tag{2}$$

Resistance between terminals 3 and 1:

- In the Δ network:

$$R_{31(\Delta)} = R_{31} + \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23}}$$

- In the Y network:

$$R_{31(Y)} = R_3 + R_1$$

Equating the two:

$$R_3 + R_1 = R_{31} + \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23}} \tag{3}$$

To find R_1:

Add Equations (1) and (3), then subtract Equation (2):

$$(R_1 + R_2) + (R_3 + R_1) - (R_2 + R_3) = \left(R_{12} + \frac{R_{31} \cdot R_{23}}{R_{31} + R_{23}}\right) + \left(R_{31} + \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23}}\right) - \left(R_{23} + \frac{R_{12} \cdot R_{31}}{R_{12} + R_{31}}\right)$$

Simplify to get:

$$R_1 = \frac{R_{12} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}$$

Similarly calculating R_2 and R_3,

$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}$$

Final Delta-to-Star Transformation Equations:

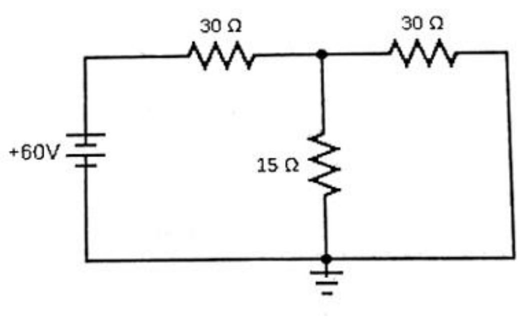
$$R_1 = \frac{R_{12} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}$$

1(d). Using nodal analysis determine the current flowing through the 15 Ω resistor.

[10 marks]

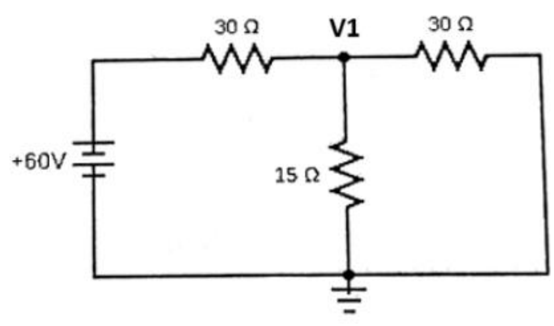


SOLUTION:

Let node be V1.

Apply Kirchhoff's Current Law (KCL) at Node V1,

$$\frac{V_1 - 60}{30} + \frac{V_1}{15} + \frac{V_1}{30} = 0$$



$$\frac{V_1 - 60}{30} + \frac{2V_1}{30} + \frac{V_1}{30} = 0$$

$$\frac{4V_1 - 60}{30} = 0$$

$$4V_1 - 60 = 0$$

$$V_1 = 15\,\text{V}$$

The current through the 15 Ω resistor is given by:

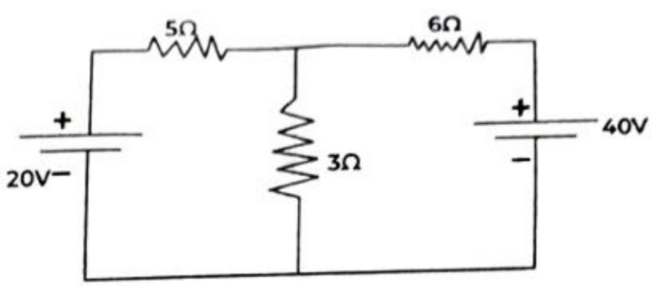
$$I = \frac{V_1}{15}$$

Substituting V_1 = 15 V,

$$I = \frac{15}{15} = 1\,\text{A}$$

The current flowing through the 15 Ω resistor is: I = 1 A

2(a). Using superposition theorem, find the current passing through the 3Ω resistor.

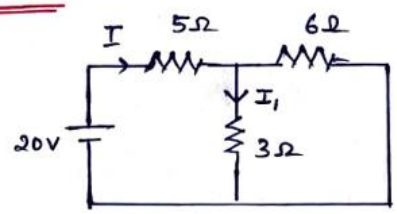


To find the current passing through the 3Ω resistor using the **superposition theorem**, consider each source (one at a time) while replacing the other sources with their internal impedances:

- Voltage sources are replaced by short circuits (0 V)

SOLUTION: **[Step 1- 4 mark; Step 2- 4 marks; Step 3 – 2 marks]**

STEP 1: Short circuit 40V source.



$$\Rightarrow \quad I = \frac{20}{5 + \left[\frac{6\times 3}{6+3}\right]}$$

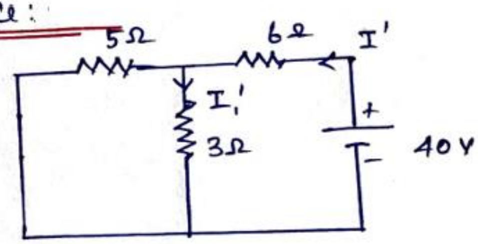
$$= \frac{20}{5 + \frac{18}{9}} = \frac{20}{5+2}$$

$$I = \frac{20}{7} \text{ A}$$

$$\Rightarrow I_1 = I \times \frac{6}{6+3} = \frac{20}{7} \times \frac{6}{9} = \frac{120}{63} = 1.90 \text{ A}.$$

$\boxed{I_1 = 1.90 \text{ A}}$ ⇒ Current through 3Ω Resistor because of 20V source.

STEP 2: Short circuit 20V Source:



$$\Rightarrow \quad I' = \frac{40}{6 + \left[\frac{5\times 3}{5+3}\right]} = \frac{40}{6+\frac{15}{8}}$$

$$= \frac{40\times 8}{48+15} = \frac{320}{63} = 5.07\text{A}.$$

$$\Rightarrow I_1' = I' \times \frac{5}{5+3} = \frac{5.07\times 5}{8} = 3.168 \text{ A}.$$

$\boxed{I_1' = 3.17 \text{ A}}$ ⇒ current through 3Ω Resistor because of 40V source.

STEP 3 : TOTAL CURRENT THROUGH 3Ω RESISTOR :

$$I_{TOTAL} = I_1 + I_1' = 1.9 + 3.17 = 5.07\ A.$$

$$\boxed{I_{TOTAL} \simeq 5\ A.}$$

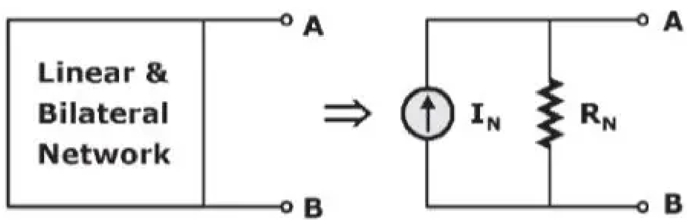
Total Current Through 3 Ω Resistor = 5 A

2(b). State Norton's theorem. Also write its step-by-step procedure.

[Statement - 4 marks, Procedure – 6 marks]

STATEMENT:

Norton's theorem states that, any two terminal network can be reduced to a current source in parallel with a resistor.



NORTON'S EQUIVALENT CIRCUIT

Where,

- I_N = Short Circuit current at terminals AB.
- R_N = Equivalent Resistance of circuit from terminals AB.
- Load Current $I_L = I_N \times [\, R_N / (R_N+R_L) \,]$

Step by step procedure for finding Norton's Equivalent:

1. Remove the load resistance and put a short circuit.

2. Find the short circuit current I_N or I_{SC}.

3. Replace all voltages with their internal resistance.

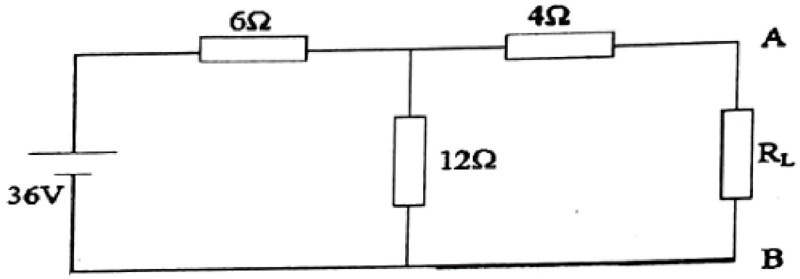
4. Find the Thevenin's looking back resistance of the network from the load terminals.

5. The current source I_N is joined in parallel with R_N between the two load terminals gives Norton's equivalent circuit.

6. Finally connect the load resistance.

Load current $I_L = I_N \times [\, R_N / (R_N+R_L) \,]$.

2(c). For the circuit shown below, find the value of R_L for which the maximum power is transferred from the source. [10 marks]



SOLUTION:

Maximum power is transferred to the load resistor (R_L) when its value is equal to the Thevenin resistance (R_{th}) of the circuit as seen from the terminals A and B.

Step 1: Simplify the Circuit to Find R_{th} :

1. Remove R_L:

To find R_{th}, disconnect R_L and determine the equivalent resistance of the remaining circuit between terminals A and B.

2. Simplify the Resistances:

- The $6\,\Omega$ and $12\,\Omega$ resistors are in parallel:

$$R_{6||12} = \frac{6 \cdot 12}{6 + 12} = \frac{72}{18} = 4\,\Omega$$

- The parallel combination ($R_{6||12} = 4\,\Omega$) is in series with the $4\,\Omega$ resistor:

$$R_{\text{total}} = R_{6||12} + 4 = 4 + 4 = 8\,\Omega$$

3. Thevenin Resistance:

Thus, the Thevenin resistance (R_{th}) is:

$$R_{th} = 8\,\Omega$$

Step 2: Maximum Power Transfer Condition

For maximum power transfer, the load resistance R_L must equal the Thevenin resistance:

$$\mathbf{R_L = R_{th} \quad \text{i.e., } R_L = 8\,\Omega}$$

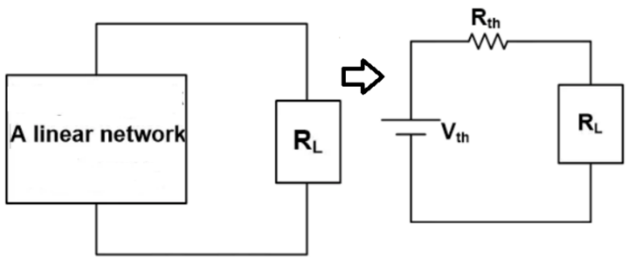
2(d). Explain about Thevenin's theorem with an example.

[Statement - 4 marks, Procedure – 6 marks]

STATEMENT:

A linear two terminal network can be replaced by a voltage source V_{th}, in series with the resistance R_{th} (or)

A linear circuit with two accessible terminals A and B can be reduced to a simple circuit as shown below.



THEVENIN'S EQUIVALENT CIRCUIT

Where,

V_{th} is the open circuit voltage at terminals AB

R_{th} is Thevenin's looking back resistance between terminals A and B.

Step by step procedure for Thevenin's equivalent circuit;

1. Remove the load whose current is required.

2. Find the open circuit voltage V_{th}, which is voltage across the two terminals where the load is removed.

3. All the voltage sources are replaced by their internal resistances.

4. Calculate the Thevenin's looking back resistance R_{th} from the two terminals.

5. Replace the entire network by the open circuit voltage V_{th} in series with the equivalent resistance R_{th}.

6. Connect the load resistance R, back to terminals A and B where it was already removed.

7. Then find the current flowing through R_L

$$I = \frac{V_{th}}{(R_{th} + R_L)}$$

3(a). Derive an expression for effective (RMS) value of sinusoidal wave in terms of its maximum value. **[Definition - 2 marks, Derivation – 8 marks]**

EFFECTIVE VALUE / ROOT MEAN SQUARE VALUE (RMS VALUE):

- When an alternating current flows through a resistance for a certain time, a certain amount of heat is produced.
- The value of **direct current** when passed through the same resistance for the same time, which produces the same heat as that of alternating current is known as Root Mean Square value or Effective Value.

RMS VALUE DERIVATION:

Sinusoidal current, $\mathbf{i = I_m \sin \Theta}$

$$\text{R.M.S. value} = \sqrt{\frac{\text{Area of squared curve}}{\text{base}}}$$

$$I_{RMS} = \sqrt{\int_0^{\pi} \frac{(I_m \sin\theta)^2 d\theta}{\pi}} \quad \text{for half cycle}$$

$$= \sqrt{\frac{I_m^2}{\pi} \int_0^{\pi} \sin\theta^2 \, d\theta}$$

$$= \sqrt{\frac{I_m^2}{\pi} \int_0^{\pi} \frac{(1-\cos 2\theta) \, d\theta}{2}}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[(\pi - 0) - \frac{(\sin 2\pi - \sin 0)}{2}\right]}$$

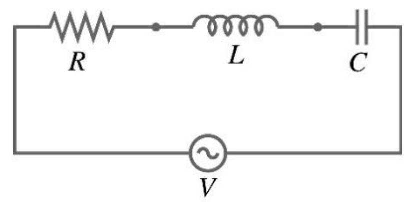
$$= \sqrt{\frac{I_m^2}{2\pi} (\pi - 0)} \qquad = \sqrt{\frac{I_m^2}{2}}$$

$\mathbf{I_{RMS} = I_m / \sqrt{2} = 0.707\, I_m}$ ➔ Where, I_{RMS} is the RMS value of sinusoidal current.

Similarly, $\mathbf{V_{RMS} = V_m / \sqrt{2} = 0.707\, V_m}$

3(b). A coil of resistance 8 Ω, an inductance of 0.1H and a capacitance of 75 μfd is connected in series across a 230 V 50 Hz supply. Find (i) current in the circuit (ii)power factor.

SOLUTION:



Given Data:

1. Resistance R=8 Ω
2. Inductance L =0.1H
3. Capacitance C=75 μF=75×10^{-6} F
4. Voltage V=230 V
5. Frequency f= 50Hz

Angular frequency:

$\omega = 2\pi f = 2\pi(50) = 314.16$ rad/s

Inductive Reactance (X_L): [1 mark]

$X_L = \omega L = 2\pi f = 314.16\times0.1 = 31.416\ \Omega$

Capacitive Reactance (X_C): [1 mark]

$X_C = 1/\ \omega C = 1/(2\pi f\ C) = 1/\ (314.16\times75\times10^{-6}) = 1\ /\ 0.023562 = 42.44\ \Omega$

Total Reactance:

$X = X_L - X_C = 31.416 - 42.44 = -11.024\ \Omega$

Impedance (Z): [2 marks]

$Z=\sqrt{(R^2+X^2)}$

$Z= \sqrt{(8^2+(-11.024)^2)} = \sqrt{(64+121.528)} = \sqrt{185.528} = 13.62\ \Omega$

Current in the Circuit (I): [3 marks]

Using Ohm's law, the current is: $I = V/\ Z = 230/\ 13.62 = 16.89$ A

Power Factor (p.f.): [3 marks]

Power factor = $\cos\phi = R\ /\ Z = 8\ /\ 13.628 = 0.587$

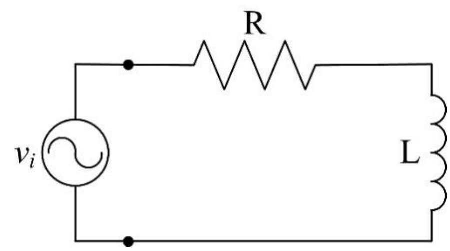
Final Answer:

(i) Current in the circuit = 16.89 A
(ii) Power factor = 0.587

3(c). A two element series circuit of R= 10Ω and X_L=15Ω has an effective voltage of 230 V at 50Hz. Determine the active power and apparent power.

SOLUTION:

Given Data:



- R = 10 Ω
- X_L = 15 Ω
- V = 230 V
- F = 50Hz

Impedance (Z): [2 marks]

$Z = \sqrt{(R^2+X^2)}$

$Z = \sqrt{(10^2+(15)^2)} = \sqrt{(100+225)} = \sqrt{325} = 18.03\ \Omega$

Current (I): [1 mark]

I = V / Z = 230 / 18.03 = 12.76 A

Power Factor (p.f.): [1 mark]

Power factor = Cos ϕ = R / Z

Cos ϕ = 10 / 18.03 = 0.5547

Active Power (P): [3 marks]

P = V I cos ϕ

P = 230 X 12.76 X 0.5547 = 1633.5 W = 1.63 KW

Apparent Power (S): [3 marks]

S = VI

S = 230 x 12.76 = 2934.8 VA = 2.94 KVA

Final Answer:

(i) **Active Power = 1633.5 W = 1.63 KW**
(ii) **Apparent Power = 2934.8 VA = 2.94 KVA**

3(d). Define the following terms: Form Factor, Peak factor, Impedance, Phase angle and Power factor.

[Each Definition - 2 marks]

FORM FACTOR:

- The ratio of r.m.s. value to the average value of an alternating quantity is called form factor.
- Form factor = R.M.S. value / Average value = 1.11

PEAK FACTOR:

- The ratio of maximum value to the r.m.s. value of an alternating quantity is called peak factor.
- Peak factor = Maximum value / R.M.S. Value = 1.414

IMPEDANCE (Z):

- It is the opposition of flow of current when a.c voltage is applied to the circuit.
- It is also defined as the ratio of voltage and current flowing through the A.C circuit, $Z = V / I$.
- Its unit is Ohm Ω.

PHASE ANGLE (φ):

- It is the angle between the voltage and current vector in an ac circuit.

POWER FACTOR (cos φ):

- It is the cosine of the angle between the voltage and current.
- It is also defined as the ratio of true power (KW) to the apparent power (KVA) in a circuit.
- $\cos \phi = KW / KVA$.

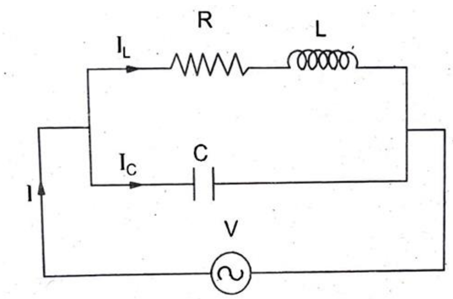
4(a). Derive an expression for bandwidth in parallel resonance circuit.

[Explanation - 3 marks; Diagram – 3 marks; Derivation – 4 marks]

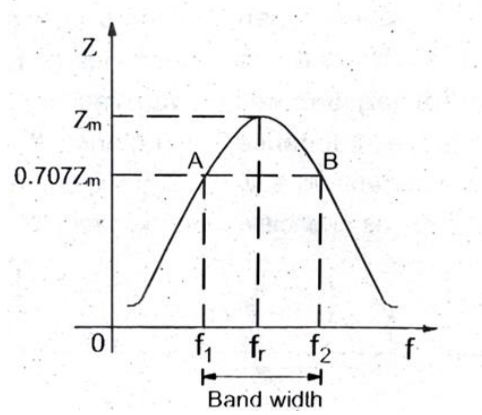
- Bandwidth of the parallel resonance is the difference between the upper and lower cut off frequencies.
- Band width = f2 – f1

  Where,
  - f_2 = Higher Cut off Frequency
  - f_1 = Lower Cut off Frequency
- f_1 and f_2 are called half power points.
- Bandwidth of a circuit is defined as the frequencies lie between two points on either side of the resonant frequency where voltage is 0.707 of its maximum value.

- It is measured between the half power points. This corresponds to the 70.7% of voltage points since power is proportional to E^2.



Two Branch Parallel Resonant Circuit



Resonance impedance diagram

DERIVATION OF BANDWIDTH OF TWO BRANCH PARALLEL RESONANCE CIRCUIT:

$$\text{Quality factor } (Q) = \frac{\text{Resonant frequency}}{\text{Bandwidth}}.$$

i.e., $$Q = \frac{f_r}{BW}$$

$$\Rightarrow BW = \frac{f_r}{Q}.$$

where, $$Q\text{-factor} = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R}.$$

$$\Rightarrow BW = \frac{f_r}{\frac{2\pi f_r L}{R}} = \frac{R}{2\pi L}$$

$$\Rightarrow \boxed{BW = \Delta f = \frac{R}{2\pi L}.}$$

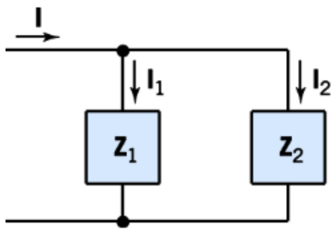
$\because$ $$\omega = 2\pi f \Rightarrow \Delta\omega = 2\pi \cdot \Delta f$$

$$\Rightarrow \Delta\omega = \frac{R}{L}.$$

Thus, BANDWIDTH of two branch parallel resonant circuit = R / L

4(b). Two impedances Z_1 = 10-j8 Ohms and Z2 = 7+j2 Ohms are connected in parallel across 230V, 50Hz supply. Find the total impedance. [10 marks]

SOLUTION:



Given:

- $Z_1 = 10 - j8\Omega$
- $Z_2 = 7 + j2\ \Omega$

When two impedances Z_1 and Z_2 are connected in parallel, Total Impedance Z_{Total} is,

$$\frac{1}{Z_{\text{total}}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

Calculating reciprocal of each impedance:

The reciprocal of an impedance Z=R+jX is :

$$\frac{1}{Z} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2}$$

For $Z_1 = 10 - j8$:

$$\frac{1}{Z_1} = \frac{10 + j8}{10^2 + (-8)^2} = \frac{10 + j8}{100 + 64} = \frac{10 + j8}{164}$$

$$\frac{1}{Z_1} = 0.06098 + j0.04878\,\Omega^{-1}$$

For $Z_2 = 7 + j2$:

$$\frac{1}{Z_2} = \frac{7 - j2}{7^2 + 2^2} = \frac{7 - j2}{49 + 4} = \frac{7 - j2}{53}$$

$$\frac{1}{Z_2} = 0.13208 - j0.03774\,\Omega^{-1}$$

Adding the reciprocals:

$$\frac{1}{Z_{\text{total}}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\frac{1}{Z_{\text{total}}} = (0.06098 + j0.04878) + (0.13208 - j0.03774)$$

$$\frac{1}{Z_{\text{total}}} = (0.06098 + 0.13208) + j(0.04878 - 0.03774)$$

$$\frac{1}{Z_{\text{total}}} = 0.19306 + j0.01104\,\Omega^{-1}$$

Calculating Z_{Total}:

$$Z_{\text{total}} = \frac{1}{0.19306 + j0.01104}$$

Multiply numerator and denominator by the conjugate of the denominator**:**

$$Z_{\text{total}} = \frac{1}{0.19306 + j0.01104} \cdot \frac{0.19306 - j0.01104}{0.19306 - j0.01104}$$

$$Z_{\text{total}} = \frac{0.19306 - j0.01104}{(0.19306)^2 + (0.01104)^2}$$

$$Z_{\text{total}} = \frac{0.19306 - j0.01104}{0.03725 + 0.00012}$$

$$Z_{\text{total}} = \frac{0.19306 - j0.01104}{0.03737}$$

$$Z_{\text{total}} = 5.17 - j0.30\,\Omega$$

Final Answer:

The total impedance is:

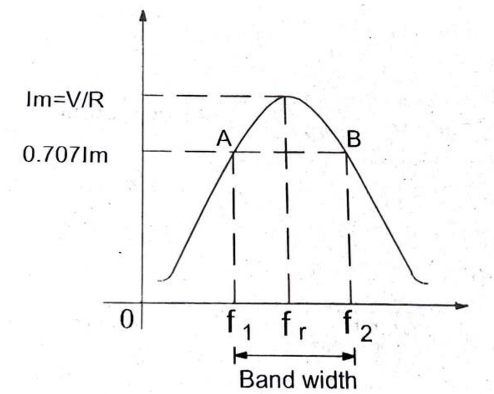
Z_{Total} = 5.17 − j0.30 Ω

4(c). Derive expressions for half power frequencies in series resonant circuit.

[Diagram – 2 marks; Derivation – 8 marks]

- At half power frequencies or cut-off frequencies the power will be equal to half the power at resonance.
- Current in the Circuit:

$$I = \frac{V}{|Z|} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



Series Resonance Curve

- The power dissipated in the resistor is:

$$P = I^2 R = \frac{V^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} R$$ **------ (1)**

- Power at resonance: $P_R = V^2 / R$ **------(2)**

- At Half power Frequency:

$P = P_R / 2$ **-------------------------(3)**

- Substitute (1) & (2) in (3),

$$\frac{V^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} R = \frac{V^2}{2R}$$

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} = \pm R$$

⇨ Multiply By ωC,

$$\omega^2 LC - 1 = \pm \omega RC$$

⇨ Divide By LC,

$$\omega^2 - \frac{1}{LC} = \pm \frac{\omega R}{L}$$

$$\omega^2 + \frac{\omega R}{L} - \frac{1}{LC} = 0$$

⇨ Solving the above quadratic equation, we get two roots,

1. $\omega = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$

2. $\omega = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$

- The half power frequency or cut-off frequencies are given by the positive roots of the quadratic equation

- **Higher Cut off Frequency**:

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

- **Lower Cut Off Frequency**:

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

- $\omega = 2\pi f$ gives,

$$f_2 = \frac{\omega_2}{2\pi} = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

$$f_1 = \frac{\omega_1}{2\pi} = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

Where**, f1 and f2 are half power frequencies in series resonant circuit**

4(d). Discuss about the effects of varying inductance and capacitance in series RLC circuit. **[Each 5 marks]**

EFFECTS OF VARYING INDUCTANCE:

- When a constant voltage, constant frequency source is applied across a series RLC circuit with inductance varied, X_C will be constant.
- The value of X_L increases directly with the value of L ($X_L = 2\pi f L$).
- When the inductance is zero, the current is limited by R and C ($Z = \sqrt{(R^2+X_C^{\ 2})}$).
- As X_L increases the effective reactance ($X_L - X_C$) reduces and the current increases.
- At $X_L = X_C$ the current is limited only by the resistance of the circuit and the current is maximum equal to (V/R). This is the resonance condition.
- Further increasing of L, the impedance of the circuit increases and the current decreases.
- When L approached infinity the current falls to zero.

EFFECTS OF VARYING CAPACITANCE:

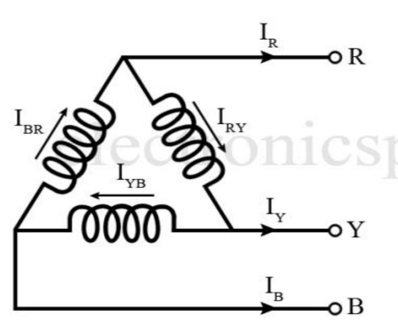
- When a series R.L.C circuit with constant frequency constant voltage source is applied across a RLC series circuit with capacitance alone varied and X_L is constant.
- X_C varies inversely with the value of C. When C is very small, X_C is very large, and the current is very small.
- As C increased, X_C decreases, when $X_C = X_L$, the current is limited, only by R and the current is maximum. This is the resonance condition.
- As C approaches infinity, X_C is equal to zero and the current is limited by R and L.

5(a). A balanced delta connected load of 4+j8 Ohms per phase is connected to a 3 phase 400V supply. Find the line current, power and total volt ampere.

SOLUTION:

Given Data:

- Load impedance per phase: Zph = 4+j8 Ω
- Supply voltage: $V_L = 400$ V



Phase Voltage:

For Delta connected load, $V_{ph} = V_L = 400V$

Impedance: [1 mark]

$$|Z_{ph}| = \sqrt{(R^2+X^2)}$$

$$|Z_{ph}| = \sqrt{(4^2+(8)^2)} = \sqrt{(16+64)} = \sqrt{80} = 8.944\ \Omega$$

Power Factor:

$\cos \phi = R / Z = 4 / 8.944 = 0.4472$

Phase Current:

$I_{Ph} = V_{ph} / |Z_{ph}| = 400 / 8.944 = 44.72$ A

Line Current: [3 marks]

$I_L = \sqrt{3}\, I_{Ph} = \sqrt{3} \times 44.72 = 77.49$ A

Active Power (P): [3 marks]

The active power is given by:

$P = \sqrt{3}\, V_L I_L \cos \phi$

$P = \sqrt{3} \times 400 \times 77.49 \times 0.4472 = 24001.0$ W = 24 kW

Volt Ampere or Apparent Power (S): [3 marks]

$S = \sqrt{3}\, V_L I_L$

$S = \sqrt{3} \times 400 \times 77.49 = 53{,}686.6$ VA = 53.69 kVA

Final Results:

- **Line Current:** $I_L = 77.49$ A
- **Power (P)**: 24 KW
- **Total Volt Ampere (S)**: 53.69 kVA

5(b). A three phase 440 V load operates with a power factor of 0.7. The total power taken from the mains is 60kW. Two-Watt meters are connected to measure the input power. Find the readings of each Wattmeter. [10 marks]

SOLUTION:

Given Data:

- Line voltage = 440V
- Total power (P) = 60 kW
- Power factor ($\cos\phi$): 0.7

Phase angle ϕ: [2 marks]

$\text{Cos}\phi = 0.7 \Longrightarrow \phi = \text{Cos}^{-1}(0.7) = 45.57°$

Line current I_{line}: [2 marks]

$P = \sqrt{3}\, V_{line} \cdot I_{line} \cdot \cos\phi$

$$I_{\text{line}} = \frac{P}{\sqrt{3} \cdot V_{\text{line}} \cdot \cos\phi}$$

$$I_{\text{line}} = \frac{60,000}{\sqrt{3} \cdot 440 \cdot 0.7} = \frac{60,000}{533.4} = 112.47 \text{ A}$$

Wattmeter Readings: [6 marks]

The readings of the two wattmeters (W1 and W2) in a three-phase system are given by:

$$W_1 = V_{\text{line}} \cdot I_{\text{line}} \cdot \cos(30^\circ - \phi)$$

$$W_2 = V_{\text{line}} \cdot I_{\text{line}} \cdot \cos(30^\circ + \phi)$$

Calculate W_1:

$$W_1 = 440 \cdot 112.47 \cdot \cos(30^\circ - 45.57^\circ)$$

$$W_1 = 440 \cdot 112.47 \cdot \cos(-15.57^\circ) = 440 \cdot 112.47 \cdot 0.9659$$

$$W_1 = 47,799.3 \text{ W or } 47.8 \text{ kW}$$

Calculate W_2:

$$W_2 = 440 \cdot 112.47 \cdot \cos(30^\circ + 45.57^\circ)$$

$$W_2 = 440 \cdot 112.47 \cdot \cos(75.57^\circ) = 440 \cdot 112.47 \cdot 0.2588$$

$$W_2 = 12,799.2 \text{ W or } 12.8 \text{ kW}$$

Final Results:

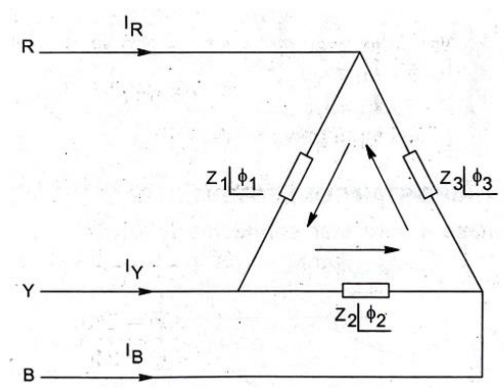
- **Wattmeter 1 Reading (W1): 47.8 kW**
- **Wattmeter 2 Reading (W2): 12.8 kW**

Note: This can also be solved by any other method.

5(c). Discuss about delta connected unbalanced loads.

[Diagram - 2 marks; Explanation – 8 marks]

- Figure shows an unbalanced delta connected load.
- Assume a phase sequence of RYB.
- The unbalanced delta connected load supplied from a balanced 3 phase supply does not present any new problems because the voltage across the load phase is fixed.
- It is independent of the nature of the load.
- For delta connection line voltage is equal to phase voltage.
- Taking V_{RY} as the reference phasor. Assuming the phase sequence is RYB.



Delta Connected Load

Line or Phase Voltages are:

$$\therefore V_{RY} = V \angle 0° \text{ V}$$
$$V_{YB} = V \angle -120° \text{ V}$$
$$V_{BR} = V \angle -240° \text{ V}$$

Phase Currents are:

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = \frac{V \angle 0°}{Z_1 \angle \phi_1°}$$
$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{V \angle -120°}{Z_2 \angle \phi_2°}$$
$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = \frac{V \angle -240°}{Z_3 \angle \phi_3°}$$

Line Currents are:

$$I_R + I_{BR} = I_{RY}$$

Thus, $\mathbf{I_R = I_{RY} - I_{BR}}$

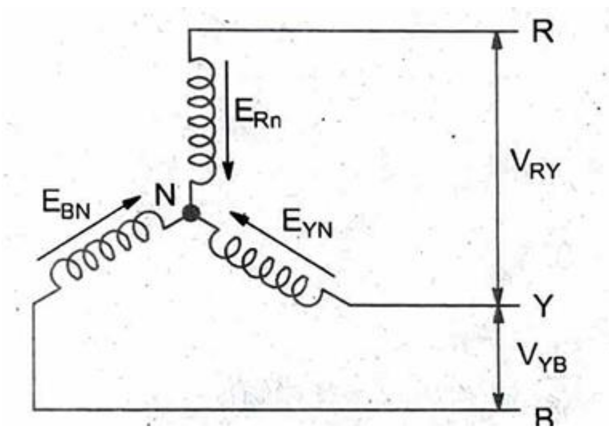
Similarly, $\mathbf{I_Y = I_{YB} - I_{RY}}$

$\mathbf{I_B = I_{BR} - I_{YB}}$

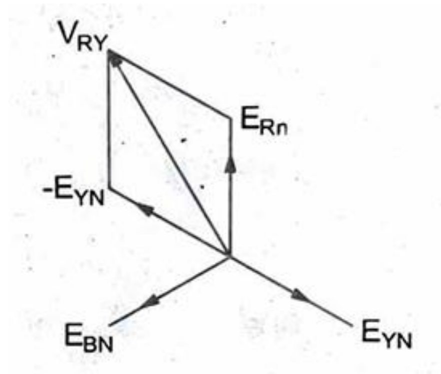
5(d). Derive the relation between voltages and currents of line and phase values in star connected systems.

[Diagram - 4 marks; Derivation – 6 marks]

- Below figure shows a balanced 3 phase Y-connected system.
- The e.m.f. generated in the three phases are E_{RN} , E_{YN} and E_{BN} are equal in magnitude but displaced 120° from one another.
  - i.e., $E_{RN} = E_{YN} = E_{BN} = V_{Ph}$
  - V_{Ph} = Phase voltage i.e., voltage between phase and Neutral
  - $V_{RY} = V_{YB} = V_{BR} = V_L$
  - V_L = Line voltage i.e., voltage between two lines.



Star Connected System



Vector Diagram

From the figure

$V_{RY} = E_{RN} + E_{NY}$ -------- (Phasor sum)

$V_{RY} = E_{RN} - E_{YN}$ -------- (Phasor difference)

From the vector diagram shown in figure,

$$V_{RY} = \sqrt{E_{RN}^2 + E_{YN}^2 + 2E_{RN}E_{YN}\cos 60}$$
$$= \sqrt{V_{Ph}^2 + V_{ph}^2 + 2\,V_{ph}\,V_{ph} \times 1/2}$$
$$= \sqrt{3}\,V_{ph}^2$$
$$= \sqrt{3}\,V_{ph}$$

Similarly,

$$V_{YB} = E_{YN} - E_{BN} = \sqrt{3}\, V_{Ph}$$

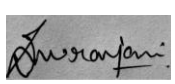
Hence in a balanced 3 phase star connection Line Voltage $V_L = \sqrt{3}\, V_{ph}$.

<u>Line Current and Phase Current:</u>

- In Y - connection each line conductor is connected in series to a separate phase as shown in figure. Therefore, line current is equal to phase current.

In Star Connection i.e., $I_L = I_{Ph}$

Prepared by,



Mrs. Manoranjani M,

Lecturer/EEE,

149, Government Polytechnic College,

Vanavasi, Salem-636457.